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CONSERVATION LAWS AND SHOCK WAVES IN THE THEORY OF RELAXATION HEAT TRANSFER

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A set of divergent forms of heat-transfer equations are presented. New laws are established that govern the behavior of the temperature field behind the front of a strong discontinuity. Comparison of theoretical and experimental data on the propagation of nonlinear waves in a sapphire crystal and liquid helium is carried out.

The study of nonlinear transfer effects in locally nonequilibrium systems continues to be a topical problem. In the present work we investigate heat transfer in a stationary medium on the basis of a Maxwell relaxation model that consists of an energy equation and a heat-flux equation:

$$c\frac{\partial T}{\partial t}+\frac{\partial q}{\partial x}+\frac{\nu q}{x}=q_{\mathrm{v}}\,,\quad \nu=0,1,2\,;\tag{1}$$

$$q+\gamma\frac{\partial q}{\partial t}=-\lambda\frac{\partial T}{\partial x}\,,\quad \lambda=\lambda(T)\,,\quad c=c(T)\,,\quad q_{\mathrm{v}}=q_{\mathrm{v}}(T,x,t)\,.\tag{2}$$

A justification of model (1), (2) and a bibliography on the subject are given in [1, 2]. The present work is a continuation of investigations [3-6] into the theory of thermal shock waves. It aims at: 1) constructing divergent forms of equations that express certain typical conservation laws of relaxation heat transfer; 2) revealing new, substantially nonlinear properties of thermal fields that contain strong discontinuities; 3) comparing theoretical calculations with experimental data on propagation of heat pulses in a sapphire crystal and liquid helium.

1. Divergent Forms of Heat-Transfer Equations. For smooth temperature fields (T, $q\in C_1$) differential equation (1) follows from the integral energy conservation law:

$$\oint_K x^{\nu}u\,dx-x^{\nu}q\,dt=-\iint_{G_K}x^{\nu}q_{\mathrm{v}}\,dx\,dt\,,\tag{3}$$

where K is an arbitrary closed piecewise-smooth contour; G_K is the region limited by this contour in the plane x, t. Any solution of Eq. (2) satisfies the following integral conservation law

$$\oint_K q\,dx-L\,dt=\iint_{G_K}(q/\gamma)\,dx\,dt\,,\tag{4}$$

$$\dot u(T)=c(T)\,,\quad \dot L(u)=w^2(u)\,,\quad w^2=\lambda/c\gamma\,,\quad V(T)=L(u)\,.$$

The conditions of dynamic compatibility across a strong discontinuity $x=x_j(t)$ of a thermal field are derived in the ordinary way [7] from Eqs. (3), (4) and have the form [3, 4]

$$N\{u\}=\{q\}\,,\quad N\{q\}=\{V\}\,,\quad N=\frac{dx_j}{dt}\,.\tag{5}$$

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The first of these relations expresses a consequence of integral conservation law (3), while the second relation is obtained from integral formula (4).

From the theory of generalized solutions for systems of quasilinear equations [7] it is known that the physical content of various phenomena can be determined by the same differential equations. The difference of these phenomena from one another is established only by discontinuity solutions, since differential equations can result from different integral conservation laws. From this viewpoint, of interest are additional (differing from Eq. (4)), conservation laws for processes of relaxation heat transfer.

The change from the integral mode of writing equations (for example, Eqs. (3) and (4)) to the differential one is made with the help of Green's formula, after which we obtain divergent equations of the form

$$\frac{\partial \Phi}{\partial t} + \frac{\partial F}{\partial x} = S , \tag{6}$$

which represent conservation laws written in terms of differential equations; the functions Φ, F, and S generally depend on T, q, x, and t. Here, following the commonly accepted approach, an extended interpretation of the term "conservation law" is applied for $S \neq 0$, since the conservation of corresponding quantities is ensured only at $S = 0$. We also assume that across the discontinuity the quantity S has no singularities of the delta-function type. The condition of dynamic compatibility, corresponding to Eq. (6), across a strong discontinuity has the form

$$N \{\Phi\} = \{F\} . \tag{7}$$

Mathematically, the hyperbolic system of quasilinear equations (1) and (2) is close to the equations of nonstationary gas dynamics, for which the problem of constructing conservation laws has been investigated in detail (a history of the problem and bibliography are given in [7-9]). Leaving aside the search for a mathematically complete set of divergent heat-transfer equations, we will indicate here the conservation laws of the type of Eq. (6) that are most interesting from the thermophysical viewpoint:

$$\text{I.} \quad \nu = 0 , \quad \Phi = uq , \quad F = uL - L_1 + \frac{q^2}{2} , \quad S = q \left(q_v - \frac{u}{\gamma} \right) , \quad \dot{L}_1 (u) = L (u) ; \tag{8}$$

$$\text{II.} \quad \nu = 0, 1, 2 , \quad \Phi = x^\nu \left(L_1 + \frac{q^2}{2} \right) , \quad F = x^\nu qL , \quad S = x^\nu \left(Lq_v - \frac{q^2}{\gamma} \right) ; \tag{9}$$

$$\text{III.} \quad \nu = 0 , \quad \Phi = BC , \quad F = \mu^{-1} \dot{B} \dot{C} , \quad B = B (q) , \quad C = C (u) ,$$

$$S = \dot{C} B q_v - C \dot{B} \frac{q}{\gamma} , \quad \ddot{B} = \mu B , \quad \mu C w^2 (u) = \ddot{C} , \quad \mu \equiv \text{const} \neq 0 ,$$

where $B(q)$, depending on the sign of the arbitrary number μ, represents a sine function or an exponential function; in an important particular case, when

$$w^2 = w_1 + 2 w_2 u , \tag{10}$$

the function $C(u)$ is expressed in terms of Bessel functions.

The class of nonlinear media that satisfy Eq. (10) comprises two variants: 1) $w_1 = 0$, i.e., the parameters $\lambda \sim u^{d_1}$, $c \sim u^{d_2}$, $\gamma \sim u^{d_3}$, which are the uniform exponential functions of temperature, $d_1 = 1 + d_2 + d_3$; for example, the values of $d_1 = 5/2$, $d_2 = 0$, $d_3 = 3/2$ correspond completely to an ionized plasma; travelling waves and discontinuities in such a medium were studied in [1]; 2) the linear dependence of the thermal conductivity coefficient on temperature

$$\lambda = \lambda_1 + 2\lambda_2 T ; \quad c, \gamma - \text{const} , \tag{11}$$

and this approximation can be applied to many substances.

For a medium with the properties contained in Eq. (10) the following conservation law is available:

$$\text{IV.}\quad \Phi = a_1 q + a_2 (u+q)^2 + a_3 (u+q)^3 - u\left[2a_2(u+q) + 3a_3(u+q)^2\right] -$$

$$- 6a_3 h + \left[2a_2 + 6a_3(u+q)\right]\left(L_1 + \frac{u^2}{2} + a_4\right);$$

$$F = LA_1 + 6a_3 L_3 + 6a_3 a_4 L;\quad A_1 = a_1 + 2a_2 q + 3a_3 q^2,\quad \dot{L}_3(u) = w^2 L_1,$$

$$h = M_1 + 2L_2 + a_4 u + \frac{u^3}{6},\quad \dot{M}(u) = uw^2(u),\quad \dot{M}_1(u) = M,$$

$$\dot{L}_2(u) = L_1,\quad S = q_v L(2a_2 + 6a_3 q) - \frac{q}{\gamma}\left[A_1 + 6a_3(L_1 + a_4)\right],$$

$$a_1, \ldots, a_4 - \text{const};$$

$$\text{V.}\quad w^2 = r^2 \exp(-2ru),\quad E = \exp(ru),\quad r \neq 0,\quad z = qE;$$

$$\text{1)}\quad |z| < 1,\quad \Phi = a_1 \arcsin z,\quad F = -a_1 r E^{-1}(1 - z^2)^{1/2},$$

$$S = a_1\left(q_v r z - \frac{q}{\gamma}E\right)(1 - z^2)^{-1/2};$$

$$\text{2)}\quad \Phi = \ln(y \pm z),\quad F = \pm ryE^{-1},\quad y = (z^2 - 1)^{1/2},\quad a_1, r - \text{const},$$

$$S = \pm z(rq_v - 1)y^{-1},$$

where the sign "+" is taken when $z > 1$ and "−" when $z < -1$.

If $\lambda(T)/c(T)\gamma(T) = w^2 \equiv$ const (in this case a thermal shock wave does not appear [3, 4]), there is the following conservation law:

$$\text{VI.}\quad \Phi = A + B,\quad A = A(q + wu),\quad B = B(q - wu),$$

$$F = w(A - B),\quad S = \dot{A}\cdot\left(wq_v - \frac{q}{\gamma}\right) - \dot{B}\cdot\left(wq_v + \frac{q}{\gamma}\right),$$

where A and B are arbitrary functions of their arguments. Relations I-VI were found with the help of the so-called direct method [8]. Conditions of dynamic compatibility (7), obtained from divergent equations I-V, can serve as a means for controlling the accuracy of numerical calculations of relaxing thermal fields with strong discontinuities.

2. The Properties of Discontinuous Thermal Fields. We will consider thermal shock waves that propagate through a uniform thermal field $u_* \equiv$ const, $q_* \equiv 0$. According to Eqs. (7) and (8), a flux of magnitude $M + (q^2/2)$ moving through a discontinuity with velocity N is equal to $P_1(N) = (q^2/2) + uL - L_1 - Nuq$; moreover, from Eq. (5) it follows that $dP_1/d(N^2) = (u - u_*)^2/2$. In the case of Eqs. (7) and (9), a flux of magnitude qL through a discontinuity is equal to $P_2(N) = qL - N[L_1 - L_{1*} + (q^2/2)]$, $dP_2/dN = q^2$.

We will write conditions (5) as:

$$q_j - q_* = N(u_j - u_*),\quad N^2 = (L_j - L_*)/(u_j - u_*) \tag{12}$$

and consider the Hugoniot line, which in the gasdynamic theory of shock waves is an analog of the Hugoniot adiabatic:

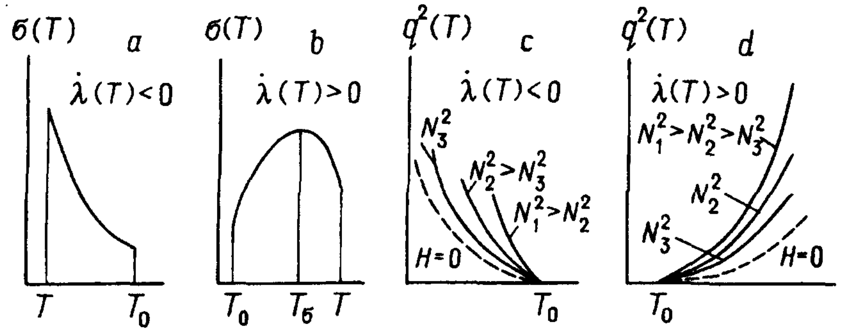

Fig. 1. Properties of thermal field behind shock waves of cooling and heating: a, b) entropy production; c, d) square of heat flux behind discontinuity front and on Hugoniot lines.

$$H\,(q,\,u;\;q_0,\,u_0) \equiv (L - L_0)\,(u - u_0) - (q - q_0)^2 = 0\,, \tag{13}$$

where H is a function of the arguments q, u; it depends parametrically on q_0, u_0. The points q, u and q_0, u_0 characterize the thermal state of the substance on both sides of the discontinuity. In the temperature range in which the speed of the propagation of thermal perturbations is a monotone decreasing function of temperature, $\dot{w}(T) < 0$, the formation of a shock wave of cooling is possible, and the formation of a shock wave of heating if $\dot{w}(T) > 0$ [3, 4]. The expression for the production of entropy in a period of 1 sec per unit volume is of the form [10]

$$\sigma = \left(q + \gamma\,\frac{\partial q}{\partial t}\right)\frac{\partial}{\partial x}\left(\frac{1}{T}\right).$$

Next, we will present the results pertaining to a one-dimensional case with plane symmetry ($\nu = 0$); in the case of cylindrical and spherical symmetries the qualitative content of the final formulas is the same. For the thermophysical parameters we take conditions (11): this variant contains basic information on the nonlinear properties of the medium, and the analytical calculations are rather simple. Using the formula obtained in [5] for $(\partial u/\partial x)_j$, behind a discontinuity front moving with a constant velocity we find

$$\sigma = \lambda q^2/[\gamma^2 u^2\,(w_0^2 - N^2)^2]\,,\quad q_0 = 0\,,\quad N = \text{const}\,. \tag{14}$$

Analysis of Eq. (14) shows that in the case of a shock wave of cooling ($\lambda_2 < 0$, $T < T_0$) the function $\sigma(T)$ is monotone decreasing (Fig. 1a), $\dot{\sigma}(T) < 0$, $\ddot{\sigma}(T) > 0$. In a shock wave of heating ($\lambda_2 > 0$, $T > T_0$), the behavior of $\sigma(T)$ is determined by the character of the function $\lambda(T)$.

Suppose approximation (11) is applied in the temperature interval $T \in [T_1, T_2]$, and, moreover, $\lambda(T_2) = b_1\lambda(T_1)$, $T_2 = b_2 T_1$, $b_1 > 1$, $b_2 > 1$; for the sake of definiteness we assume that $T_1 = T_0$. It turns out that $\sigma = \sigma(T)$ is the nonmonotonic function that has a maximum at $T = T_\sigma$:

$$T_\sigma/T_0 = 2b_3\,(1 - 2b_3)/(1 - 3b_3)\,,\quad b_3 = (b_1 - b_2)/(b_1 - 1)\,.$$

If the thermophysical properties of the medium are such that $T_\sigma < T_0 = T_1$, then in the temperature region behind the shock wave front the function $\sigma(T)$ is a monotone decreasing function; when $T_\sigma > T_2$, the function $\sigma(T)$ increases monotonically. Moreover, if the number b_1 satisfies the inequalities $(4b_2 - 1)/3 < b_1 < (3b_2 - 1)/2$, then we have $T_0 < T_\sigma < T_2$. Consequently, in this variant the entropy production is an inmonotonous function of temperature (Fig. 1b). The function $q^2(T)$ on the Hugoniot line in a shock wave of heating is a monotone increasing function, whereas in the case of shock cooling it is a monotone decreasing function; in both cases $d^2(q^2)/dT^2 > 0$. At the point $T = T_0$ the Hugoniot lines touch the T axis; in Figs. 1c and 1d these are the dashed curves; shown also by solid curves is the family $N^2 = \text{const}$.

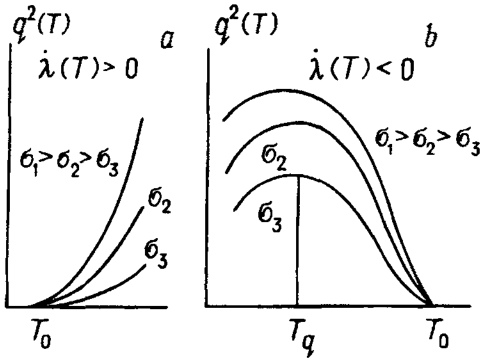

Fig. 2. Isolines of entropy production behind shock wave front: a) heating; b) cooling.

Now, we will consider the family of isolines $\sigma = \text{const}$. In a wave of heating the function $q^2(T)$ is a monotone increasing function; it is convex downward (Fig. 2a). The behavior of the σ-lines in a wave of cooling is illustrated by Fig. 2b. Assume that

$$T \in [T_1, T_2]\,,\ \ \lambda(T_1) = d_1 \lambda(T_2)\,,\ \ T_1 = d_2 T_2\,,\ \ d_1 > 1\,,\ \ 0 < d_2 < 1\,,\ \ T_0 = T_2 > T\,.$$

Analysis of formula (14) shows that on the line $\delta = \text{const}$ the function $q^2(T)$ has a maximum at the point $T = T_q$:

$$d_2 < \frac{T_q}{T_0} = \frac{1}{6}\left[1 + 2d_3 - (4d_3^2 - 8d_3 + 1)^{1/2}\right] < 1\,,\ \ d_3 = \frac{2(d_1 - d_2)}{(d_1 - 1)}\,,$$

and, these formulas have a physical meaning if the nonlinear properties of the medium are such that: 1) $0 < d_2 \le 1/2$, $d_1 > 1$; 2) $1/2 \le d_2 < 2/3$, $d_1 > d_2/(2 - 3d_2)$. Under these conditions on the line $\sigma = \text{const}$ in the region behind a wave cooling front the function $q^2(T)$ is nonmonotonic. In the class of media (12), when $\lambda_2 < 0$, a monotonic variant is absent for $q^2(T)$ on the δ-line.

3. Heat Pulse in a Sapphire Crystal. The temperature dependence of heat pulses in sapphire is investigated experimentally in [11]. We will consider the evolution in time of a pulse received at the isothermal boundary of the crystal for temperatures of 6, 18, and 23 K, beginning from $t = 1.5$ μm, when the image of the normalized amplitudes of the pulses observed becomes distinct. It turns out that the specific features of the change with temperature in the pulse shape obtained in the experiment are described qualitatively by an exact solution [12] of heat-transfer equations (1) and (2):

$$x(u, \tau) = \frac{f_1}{(u + k_1)} + \dot{\zeta}(\omega)\,,\ \ \omega = \frac{(u + k_1)}{\tau}\,,\ \ \tau = \exp(-t/\gamma)\,,$$

$$\gamma q(u, \tau) = f_1 - \tau\zeta(\omega) + (u + k_1)\dot{\zeta}(\omega)\,,\ \ a = a_1 (u + k_1)^{-2}\,,\ \ a_1\gamma = f_1^2\,,$$

$$a_1, k_1, \gamma - \text{const}\,,$$

where $\zeta(\omega)$ is an arbitrary function. In fact, let the heat flux $q_0(\tau) = q(u_0, \tau)$ on the isotherm $T_0 \equiv \text{const}$, $u_0 = u(T_0)$ be known from the experiment. Then, we can easily find the function

$$\zeta(\omega) = \frac{\omega}{(u_0 + k_1)}\left[\zeta(u_0 + k_1) + \int\limits_{u_0 + k_1}^{\omega} (\gamma Q_0 - f_1)\, d\omega\right],$$

where the dependence $Q_0 = Q_0(\omega)$ is obtained from $q_0(\tau)$ by replacing τ by the argument $(u_0 + k_1)/\omega$. When $T \in [T_0, T_1]$, the heat flux has the form

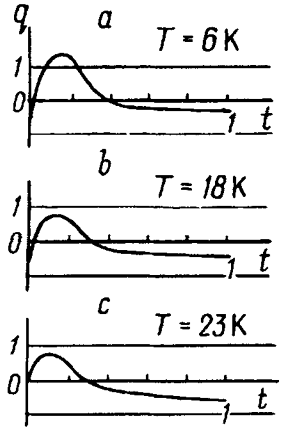

Fig. 3. Shape of the heat pulse on isotherms in sapphire crystal.

$$q = \frac{f_1}{\gamma}\left(1 - \frac{u + k_1}{u_0 + k_1}\right) + \frac{(u + k_1)}{(u_0 + k_1)} q_0(\vartheta), \quad \vartheta = \frac{(u_0 + k_1)\tau}{(u + k_1)}. \tag{15}$$

In order to obtain an expression for $q(u, \tau)$, when $T \in [T_1, T_2]$, we proceed in a similar way. It is only required to redetermine the parameters a_1, k_1, f_1, which characterize the thermophysical properties of the medium in this temperature interval. As a result, we have

$$q = \frac{f_{11}}{\gamma}\left(1 - \frac{u + k_{11}}{u_1 + k_{11}}\right) + \frac{(u + k_{11})}{(u_1 + k_{11})} q_{01}(z), \quad z = \frac{(u_1 + k_{11})\tau}{(u + k_{11})}, \tag{16}$$

where $u_1 = u(T_1)$; k_{11}, f_{11} are the new values of the constants. The function $q_0(\vartheta)$ should be written separating explicitly the argument τ, which subsequently is replaced by z; this gives the function $q_{01}(z)$.

Let us give an example. We take temperature $T_0 = 6$ K as the initial one. The shape of the heat pulse on this isotherm will be taken in a form similar to the experimental one (Fig. 3a): $q_0(t) = A_2 T \exp(B_2 t) - A_1 t \exp(B_1 t)$, $t \geq 0$; $A_1 = 108.7$; $B_1 = 40$; $A_2 = 40.77$; $B_2 = 10$. As the scales of the thermophysical parameters for dedimensionalization, we use their values at $T = 20$ K. The absolute value of the number k_1 should be rather large; when $c \sim T^{\alpha}$, this makes it possibile to assume for the thermal conductivity coefficient that $\lambda \sim T^{\alpha}$, i.e., that $\lambda / c \sim$ const. When $\gamma \equiv$ const, in this class of solutions we obtain a velocity of propagation of thermal perturbations that is nearly constant, $w^2 \sim$ const, which is satisfied for sapphire [11] at temperatures of from 4 to 40 K. Further we assume that $\alpha = 3$. In the range of from 6 to 18 K we take $k_1 = 5$, $f_1 = 7.4566$, and $\gamma = 1$; the results of calculation by formula (15) give the expression

$$q_1 = -0.2386 + 1.032\left[(A_1 \vartheta^{B_1} - A_2 \vartheta^{B_2}) \ln \vartheta - 0.2\right], \quad \vartheta = 0.96863\tau,$$

and these results are shown in Fig. 3b. For the second temperature interval of from 18 to 23 K, we must take $k_{11} = 90$, $f_{11} = 90.25$, $\gamma = 1$, $A_{11} = 30.38$, $A_{21} = 29.64$, and $z = 0.996981\tau$,

$$q_2 = -0.2733 - 1.003\left[0.2 + (0.035 + t)(A_{11} z^{B_1} + A_{21} z^{B_2})\right].$$

This relation is shown in Fig. 3c. The influence of temperature on the shape of the heat pulse obtained theoretically is similar to that observed in the experiment: as the temperature increases, 1) the peak of the maximum decreases, and 2) on attainment of the maximum, the passage of $q(t)$ through zero occurs more rapidly.

4. Heat Pulse in He II. Nonlinear second-sound waves in liquid helium have been investigated theoretically and experimentally in many works (a history and bibliography of this problem are given in [13, 14]). In the context of the hyperbolic heat conduction equation, some problems of the propagation of thermal perturbations in He II are stated in [15]. We will consider two series of experiments [16-18] with shock waves in He II. In [16], using

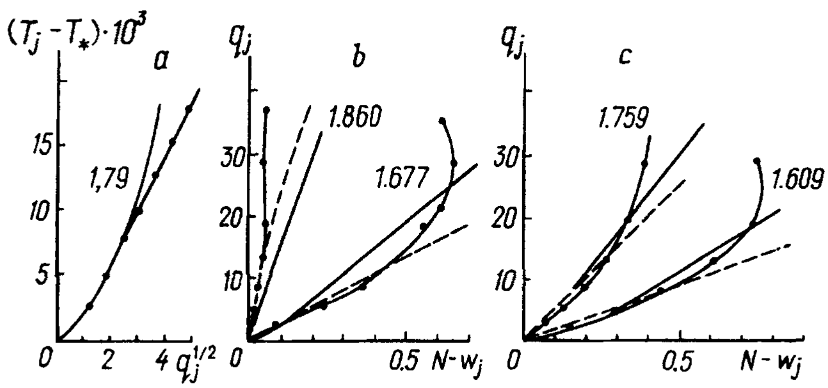

Fig. 4. Second-sound shock wave in He II, comparison with experiments: a) [18]; b, c) [17]. The solid curves denote calculation by formulas (17); the dashed curves denote the theoretical relation of [16]. T, K; q, W/cm^2; N, m/sec.

the Burgers equation, the relationship between the radiating pulse power and the shock wave velocity is calculated, and comparison with the experiments of [17] is performed. The dependence of the temperature pulse amplitude on the radiator power is determined in [18].

We will describe the propagation of second sound that represents a temperature wave [13, 14] by heat-transfer equations (1) and (2). In the experiments of [16-18], a second-sound pulse was excited and a strong discontinuity in the temperature field appeared that moved from the source of perturbations to the signal detector. We simulate this strong discontinuity at $t = 0$ by conditions of dynamic compatibility (12), to which it is necessary to add the stability condition $w_*^2 < N^2 < w_j^2$ [6, 7]. Then the heat flux q_j should be interpreted as the power of the radiated pulse and $(T_j - T_*)$ as the initial amplitude of the temperature pulse that propagates through the uniform background $T_* \equiv \text{const}$, $q_* \equiv 0$.

The thermophysical properties of helium are taken from [19]. The units of measurement are: sec, J/cm$^3\cdot$deg; λ/γ, W/cm$\cdot$sec$\cdot$deg; q, W/cm^2; N, m/sec; T, K. In performing a comparison with the experiments of [16-18], the following should be kept in mind. For He II in the case of second sound, shock waves of heating are observed when $1 < T < 1.88$ K. This range is wider than the region $T \in (1.2; 1.65)$ of a monotonic increase in the function $w^2(T)$, where, according to model (1), (2), and (12), the formation of shock waves of heating is possible. Therefore, in order to coordinate model (1), (2), and (12) with the experimental data of [17, 18], we will take into account that λ/γ increases monotonically when $T \in (1.2; 1.95)$ and assume that $u_j - u_* = \tilde{c}(T_j - T_*)$, $\tilde{c} \equiv \text{const}$. If we admit, for example, that at $T \in [T_*, T_j]$ the heat capacity $c = \rho c_p$ depends linearly on temperature, then $\tilde{c} = (c_* + c_j)/2$. The remaining parameters, namely, $w^2 c \equiv \lambda/\gamma = \kappa_0 T^{n_1}$, w_*^2, will be taken from the experimental data of [19].

Then Eq. (12) will take the form

$$q_j = \tilde{c}\,(T_j - T_*)\,N\,,$$
$$N^2 = \kappa\,(T_j^{1+n_1} - T_*^{1+n_1})/[\tilde{c}\,(T_j - T_*)]\,, \quad \kappa = \frac{\kappa_0}{1+n_1}\,, \tag{17}$$

where $\tilde{c}$ serves as the parameter of agreement between the theoretical and experimental data. We also note that formulas (17) do not contain exlicitly a numerical value for the time of thermal relaxation. Figure 4a presents a comparison at $T_* = 1.79$ K of calculations performed on the basis of Eq. (17) (solid curve) with the experimental values of [18] (points). Calculations were carried out at $\tilde{c} = 0.41$, $w_* = 19.947$. If $T \in [1.75; 1.80]$, we have $n_1 = 3.9443$; $\kappa = 33{,}148.006$; $c^1 \in [0.3655; 0.4211]$. For ease of representation, here and below we indicate the interval in which the experimental value of the heat capacity c^1 changes. In Figs. 4b and 4c the experimental data of [17] at four values of T_* are marked by points, theoretical calculations from [18] are represented by dashed curves, and calculations by the formulas of [17] are represented by solid curves. The calculations were carried out at the following values of the parameters:

1) $T_* = 1.609$; $\tilde{c} = 0.2365$; $w_* = 20.3784$; $\kappa = 13{,}332.8$; $n_1 = 5.2192$; $T \in [1.60; 1.65]$, $c^1 \in [0.2322; 0.2716]$;

2) $T_* = 1.677$; $\tilde{c} = 0.2932$; $w_* = 20.4$; $\kappa = 16{,}502.672$; $n_1 = 4.8989$; $T \in [1.65; 1.70]$, $c^1 \in [0.2716; 0.3161]$;

3) $T_* = 1.759$; $\tilde{c} = 0.3725$; $w_* = 20.157$; the remaining numbers are indicated in the legend of Fig. 4a;

4) $T_* = 1.860$; $\tilde{c} = 0.4982$; $w_* = 19.27$; $\kappa = 110{,}290.48$; $n_1 = 2.5172$; $T \in [1.85; 1.90]$, $c^1 \in [0.4835; 0.5535]$.

As seen from Fig. 4, there is a fairly good (from the viewpoint of the adopted model representations of the process) quantitative agreement between the theoretical and experimental results. The calculated curves obtained have the same qualitative properties as those observed in experiments: 1) a monotonic increase in $T_j - T_*$ with an increase in q_j (Fig. 4a); 2) as the background temperature T_* increases, the dependence of q_j on $N - w_*$ becomes steeper (Figs. 4b and 4e); in this case these curves touch the horizontal axis and are convex downward.

Conclusions. We obtained a thermophysically informative set of divergent forms of equations in terms of which it is possible to write one-dimensional equations of relaxation heat transfer. For thermal shock waves of heating and cooling we established the characteristic features of the behavior of the Hugoniot lines and of the isolines of entropy production, as well as the dependences of heat flux on temperature. A comparison is made of theoretical calculations with experiments known in literature on the propagation of waves in nonlinear media, and the efficiency of the Maxwell model of relaxation heat transfer is shown.

NOTATION

T, temperature; q, specific heat flux; t, time; x, Cartesian (radial) coordinate; λ, thermal conductivity coefficient; c, specific volumetric heat capacity; q_v, power of internal heat sources; γ, time of heat flux relaxation; w, velocity of heat propagation; $\nu = 0, 1, 2$, parameter characterizing the symmetry type (plane, cylindrical, and spherical); the braces denote the difference between the values of the quantity enclosed in the braces on both sides of the discontinuity. Subscripts and superscripts: dots above the symbol of the function denote ordinary differentiation with respect to its argument; $*$, parameters of the background through which the shock wave propagates; j, values of the functions behind the discontinuity front.

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